of the group, and summing the corresponding coefficients k_i. Application of such a method to example 3 showed that the accuracy of the solution remained satisfactory, while the time necessary was reduced insignificantly.

—*Received February 2, 1961*

Reference

[1] Gurin, L. S., "An experiment applying the Monte Carlo method for the location of extreme values of a function," *Computational Mathematics and Computational Technology* (Mašgiz, Moscow, 1962).

Reviewer's Comment

The problem discussed here has been relegated mostly to mathematical recreations in the western literature, although it has appeared in serious (oral) discussions as a "warehouse location" problem: given n points in the plane, find m points—"centers" or "distribution centers"— such that, if each of the points is joined to some center by a line segment, the sum of the lengths of the segments is minimized. An account of the problem, due to Steiner, for $m = 1$ is given in *What Is Mathematics* by Courant and Robbins (Oxford University Press, New York, 1941), in which it is remarked that the general problem does not lead to interesting (theoretical) results. However, this problem remains useful in applications.

The problem is a discrete-nonlinear programming problem and could, in principle, be solved exactly by some recent extensions of current methods for discrete programming problems. The discrete feature is handled here, however, in what is probably the best practical way, by the "Monte Carlo" device of selecting a group of centers at random and assigning each point to the nearest center. The nonlinear portion is handled by an ordinary gradient method, although the authors' device of insuring that the gradient direction always changes by at least 45° so that "a point which is not the solution of the problem cannot be approximated arbitrarily closely" is new. Since, however, the function being minimized is convex, it does not seem that that difficulty could arise here, but the device is interesting in general. It is unfortunate that evidence for a reduction of solution time is not cited, and that the computer used is not described so that the meaning of the cited solution times can be understood.

The problem of finding a single center has a neat mechanical analogue. It is the equilibrium position of a point acted on by n unit forces, each of magnitude k_i directed toward the ith given point. A system of string, weights, and pulleys embodying this principle would probably be an effective alternative means of solving this problem.

—PHILIP WOLFE
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NOVEMBER 1963 AIAA JOURNAL VOL. 1, NO. 11

Digest of Translated Russian Literature

The following abstracts have been selected by the Editor from translated Russian journals supplied by the indicated societies and organizations, whose cooperation is gratefully acknowledged. Information concerning subscriptions to the publications may be obtained from these societies and organizations. Note: Volumes and numbers given are those of the English translations, not of the original Russian.

APPLIED MATHEMATICS AND MECHANICS (*Prikladnaia Matematika i mekhanika*). Published by American Society of Mechanical Engineers in conjunction with Pergamon Institute

Volume 26, Number 2, 1962

Nonsteady Propagation of Cracks, G. I. Barenblatt, R. L. Salganik, and G. P. Cherepanov, pp. 469–477.

Investigations of the processes of crack propagation have continued now for a considerable period of time, and it would be fair to say that in the field of stationary propagation of cracks the investigations have more or less reached completion.

One of the simplest problems of nonstationary propagation of cracks would appear to be the problem of the widening at constant velocity of a rectilinear crack in a uniform stress field perpendicular to the line of the crack. This problem has been investigated by a number of authors, starting with Mott, but it was not until the paper by Broberg that it was treated as a problem of the dynamic theory of elasticity. Broberg, however, neglected the effect of cohesive forces, and for this reason came to the conclusion that the uniform propagation of cracks can take place only at a velocity equal to the velocity of propagation of Rayleigh surface waves: at any other velocity an uncompensated singularity occurs in the stress field at the end of the crack.

The present paper investigates on the basis of certain assumptions the effect of cohesive forces and derives an equation which defines the velocity of propagation of a crack in terms of the applied stress. It is shown that for every material there is a certain minimum velocity of stable uniform crack propagation. It is also shown that the velocity of stable propagation of a crack increases with increase in the splitting force and tends to the Rayleigh velocity: it would appear that in isotropic bodies the formation of a regime of uniform propagation at the Rayleigh velocity is prevented by the occurrence of branching of the crack.

Formulation of Refined Theories of Plates and Shells, I. G. Teregulov, pp. 495–502.

The attempt to refine theories of plates and shells was started in other studies, and at the present time many papers are devoted to this problem. These papers usually use one of a number of assumptions. A survey of them is beyond the scope of this note. We mention only the papers in which a certain error is specified at the outset, for example, of the order of h^4/L^4 compared to unity ($2h$ is the thickness and L is the transverse dimension of the plate) and the differential equations corresponding to this accuracy are obtained. Boundary conditions to within this error have been obtained and presented elsewhere. In the present author's view certain references are of the greatest interest in regard to the methodology of formulating a theory, even though they cannot lay claim to consistency in the question of refinement.

We present a quite general method of formulating refined theories of plates and shells which goes back to Reissner's work for its ideas. It is based on a generalized variational principle of the nonlinear theory of elasticity.

Motion of a Gyroscope Supported by Ball Bearings on Gimbals, S. A. Kharlamov, pp. 529–536.

In the theory of a free gyroscope, it is assumed that the location of its instantaneous axis of rotation relative to the inner ring of the gimbals is fixed in the coordinate system attached to this ring. This paper considers the motion of a gyroscope on gimbals for which the stated assumption is not made. The relative motion of the gyroscope is determined purely kinematically by means of prescribing the motion of its center of inertia and the orientation of the figure axis. The inertia forces of the relative motion generate small oscillations of the gyroscope in the neighborhood of the stationary motion causing it to precess relative to the axis of the outer ring. Such a relative motion can take place if the gyroscope is supported by ball bearings on gimbals.

Remarks Concerning the Paper "On Some Optimum Rocket Trajectories" by G. Leitmann, A. I. Lur'e, pp. 576–577.

AUTOMATION AND REMOTE CONTROL (Avtomatika i Telemekhanika). Published by Instrument Society of America, Pittsburgh, Pa.

Volume 22, Number 12, December 1961

Isoperimetric Problem in Analytic Design, I. A. Litovchenko, pp. 1417–1423.

Analytic design of an optimum controller is considered in the presence of an isoperimetric constraint and with saturation. Special points of the problem are illustrated by a simple example.

The problem of analytic design of controllers was solved in another study under the condition that the modulus of deviation of the regulating member is bounded. But our attention had previously been drawn to the importance of the integral constraints imposed on the coordinates and/or on the speed of the controller. In the present work an attempt is made to clarify the role of such constraints in the structure of an optimum regulator such that the integral-square error of the system is minimized.

Summary:

1) If the initial deviations of the system satisfy the inequality derived, the optimal phase trajectory is determined only by the restriction noted because when the inequality takes place, the trajectory is always within the boundary $|\xi| = \xi$.

2) But if there is no constraint as given, the optimal trajectory also contains the boundary regions for which $|\xi| = \bar{\xi}$, as shown in detail in our simple example.

Note: The optimal trajectory in this text is such that it satisfies only the necessary conditions of optimality, although, strictly speaking, it would be pointless to use the formula without a prior investigation of the sufficient conditions.

Some Approximate Methods for Solving Problems of Optimal Control of Distributed Parameter Systems, A. G. Butkovskii, pp. 1429–1438.

Approximate methods for solving problems of optimal control of distributed parameter systems are studied: the differential-difference methods and also the method of moments.

Investigation of Nonlinear Unsteady-State Systems Which Are Acted upon by Discontinuous Random Disturbances, M. I. Gusev, pp. 1455–1462.

The present article is concerned with the investigation of nonlinear unsteady-state automatic control systems which are acted upon by discontinuous constraining random and determinate forces. The application of digital and combined digital and analog computers for determining the distributions of generalized coordinates is considered for the class of automatic control systems under investigation.

Synthesis of Relay Systems from the Minimum Integral Quadratic Deviation, Chan Jên-Wei, pp. 1463–1469.

One simple method is given for finding the optimal control law in the form of a function of the phase coordinates.

Dynamics of Photoelectric Compensators, A. N. Tkachenko, pp. 1530–1537.

The presence of high amplification (up to 10^8 in voltage) and low zero drift always have brought a broader recognition in measurement techniques and automation to the d.c. photoelectric compensator. The wide use of these instruments has caused the Leningrad electrical measuring instrument plant "Vibrator" to begin the series production of photoelectric compensators combining a galvanometer, optical system, and photoresistance into a single design unit in 1957.

However, in the construction of specific circuits containing the photoelectric compensator, a number of difficulties are frequently encountered. Due to presence of feedback, self-oscillation can arise in the circuit. It is difficult to achieve a satisfactory rate of response in a stable circuit. The problem of synthesizing photoelectric compensator circuits with prescribed dynamic properties is therefore of great interest.

The method of calculation here is based on the assignment of the roots of the characteristic equation, according to T. N. Sokolov, and permits the synthesis of photoelectric compensator circuits with both stiff and flexible negative feedbacks.

Conclusions: The synthesis of photoelectric compensators with prescribed response rate can be realized using prescribed distribution in the complex plane of the roots of the characteristic equation of the closed-loop system. This method leads most simply to design formulas for the stabilization network parameters.

Those stabilization circuits for the photoelectric compensator can be recommended which use an integrating network at the input to the vacuum amplifier (circuits with stiff and with flexible feedback) or a differentiating network in the galvanometer circuit (circuits with stiff feedback).

Method of Solution of Multiple-Loop Sampled Data System Equations, I. M. Burshtein, pp. 1546–1550.

The simultaneous existence in pulsed systems of continuous and discrete signals makes it very difficult for us to obtain the necessary transforms. In order to obtain the discrete Laplace transform at the output of a single-loop pulsed system, G. R. Stibitz and C. E. Shannon introduced several severe limitations upon the transients in the forward and feedback points. An analogous problem for single-loop systems, where no additional limitations were imposed, was solved by Ya. Z. Tsypkin by applying the external action to the output of the pulsed elements (PE). Later, D. Ragazzini and L. Zadeh compiled a table of transforms of output signals for typical single-loop systems containing one or several PE. Ya. Z. Tsypkin obtained a formula for the discrete transform of the signal at the output of a single-loop multiple circuit system with several nonsynchronous PE. In order to find the a-transform at the output of a single-loop system with several PE where the closure period was an integral ratio G, Kranc made use of the structural transformation to reduce the program of closed and open intervals of the PE to a single period. However, the solution of the equations obtained for the multiple-loop system was obtained in a very complex manner, by expanding the transforms in infinite series using a special formula. The method of writing the equations for a multiple-loop system with several nonsynchronous PE is indicated by Ya. Z. Tsypkin. It is assumed that, by using open-loop branches at the inputs to the PE, the initial system becomes an open-pulsed system with a common continuous part and an equal number of inputs and outputs; therefore we consider that the external action is referred to the inputs of the PE.

In the present paper we consider the problem of determining, by algebraic means, the z-transforms for a sampled-data system of arbitrary structure with many PE, each of which opens and closes according to an individual program. There is a general repetition rate which is uniform for the system.

The structure of the system is determined by the number of loops which interconnect, in a definite manner, the various branches of the direct (forward) path; each branch may be paralleled by its own feedback loop. In each loop there is an algebraic addition of the signals which arise from within the system or from other branches of the system; the resulting signal is transmitted to all the branches leaving the point. The branches of the linear pulsed system can contain amplifiers, continuous filters, delay elements, PE, discrete filters, and devices for converting discrete signals into continuous ones.

Volume 23, Number 1, January 1962

Critical Case of Absolute Stability, V. M. Popov, pp. 1–21.

In this article, the stability of the trivial solution of a class of systems of differential equations describing the behavior of certain automatic systems with single nonlinear elements is investigated. The systems studied are of the same type as those considered earlier by A. I. Lur'e and are characterized by the property that on replacing the nonlinear element by a linear one, a system of differential equations with constant coefficients is obtained whose characteristic equation has two zero roots. Sufficient criteria for the absolute stability of the trivial solutions of such systems have already been established, as is known, with the aid of the Lyapunov second method and other methods. A new method is used here for this same purpose; this has been applied earlier by the author to the case of a system of differential equations of various types containing one or several nonlinearities.

Passage of Random Signals through a Time Discriminator and an Integrating Amplifier II. Correlation Functions and Spectral Densities of Output Signals of Pulse Systems, F. M. Kilin, pp. 22–30.

The recurrence relationship obtained in another paper for the coordinate lattice function $\Phi[n]$ is applied to investigate statistical dynamics of a pulse system and to determine the correlation functions and spectral densities of the output signals of such pulse systems. Properties of a pulse system are found by analyzing the expressions representing random processes which occur in a given system.

Investigation of a Gradient System of Automatic Optimization with Random Interference at the Input and Output of the Object, Ts. Ts. Paulauskas, pp. 31–40.

An investigation is carried out of an m-dimension gradient system of optimization in the presence of random interference at the input and output of the object, in the case when a discrete optimizer is used in the elements. Methods are given for determining the error in the steady search regime. The results are illustrated by examples. The influence of interference on the system is studied.

Reliability of Remote Control Systems, V. A. Lutskii, pp. 104–108.

A method of evaluating the reliability of remote control systems is presented by introducing block-weight coefficients depending on the internal structure of the device. A special "vitality" index is introduced.

Supplement to "L. S. Pontryagin's Maximum Principle and the Optimum Programming of Rocket Thrust," V. K. Isayev, pp. 114–117.

Volume 23, Number 2, February 1962

Synthesis of Automatic Control Systems with Random Inputs, N. I. Sokolov, pp. 126–135.

The paper proposes a method for determining the desired transfer function for an automatic control system with a specified astaticism order for random stationary inputs. The synthesis of linear automatic control systems can be subdivided into two successive stages: a) determining the desired transfer function for the automatic control system from the condition governing the satisfaction of the engineering requirements when the system is in the closed-loop state; b) determining the block diagram and the parameters of a linear automatic control system from the desired transfer function.

The following problems are solved during the first stage of synthesis.

1) From the functionally necessary elements (i.e., elements without which an indirect control system cannot operate, such as the control object, the actuating element, the amplifier, the detecting unit) a block diagram for the automatic control system is formulated and the degrees of polynomials in the denominator and numerator of the transfer function for this system are determined.

2) From the condition governing satisfaction of the specified engineering conditions (the astaticism order, the minimum mean-square error, the allowed overregulation, the duration of the transient response) we determine the desired transfer function for the automatic control system.

During the second stage of synthesis, the overall gain of the system in the open-loop state is determined, as well as the circuits and parameters of the corrective networks and the points at which they are connected. This is achieved on the basis of the condition requiring that the transfer function of the corrected system correspond to the desired transfer function.

In this paper we shall study only the first stage of synthesis.

Method of Calculating the Complex Roots of Algebraic Equations by Means of Analog Computers, A. A. Kosarev, A. V. Martynov, and L. I. Yakunina, pp. 149–154.

We consider the equation

$$x^n + a_1x^{n-1} + \ldots + a_n = 0 \tag{1}$$

with real coefficients. Several specialized devices have been described for the determination of the roots of Eq. (1). One class of such methods is given, for example, in another paper. However, despite the existence of these specialized root calculators, it is of great interest to obtain methods for the solution of algebraic equations that employ the ordinary, widely distributed analog computers.

In the literature, methods are described for solving Eq. (1) which use general purpose analog computers. The majority of these methods are based on the investigation of the dynamic system having (1) as its characteristic equation, on the reduction of (1) to a system of two equations, or on obtaining graphs of polynomials. These methods have their advantages and disadvantages.

The existing methods of calculating roots of Eq. (1) using general purpose analog computers can be conveniently used only to calculate the real roots. The methods given in the literature for determining the complex roots suffer from several serious disadvantages. For example, they do not obtain all the roots, but only those roots having the largest real parts. The calculation of the other roots requires translations and rotations of the roots of the polynomial, which means that the computer must be readjusted. The method proposed in another paper can be used only in special cases (when the imaginary parts of the roots are small compared with the real parts or vice versa) to obtain approximate values of the complex roots. In the general case, this method will yield only very rough results.

In the plan proposed, all the complex roots are obtained without any translation or rotation and consequently without readjustment of the apparatus, and the values obtained for the roots have reasonably good accuracy. Further, questions that are important in connection with simulation, such as that of the scaling of the variables, have hardly been considered in the literature. Certain recommendations are given in another paper concerning the choice of scaling transformations for a series of special cases, but no general method is given for choosing the scaling. The absence of any such general method leads to the necessity, during the solution, of changing the scaling of the variables, and this means that, if good accuracy is desired, there is a considerable increase in machine time for the solution due to the many readjustments necessary.

In the present article, we propose a method of determining the complex roots of Eq. (1) by using a general purpose computer and also give a method for finding the boundaries of the region in which these roots are located. This latter method can be used in scaling. The method we propose for calculating the complex roots requires the preliminary determination of the real roots, and so we also describe one of the ways of finding real roots with an analog computer.

Conclusions: In the proposed method for determining the complex roots of Eq. (1), the question was considered of obtaining sufficiently accurate bounds for the region in which the roots lie, so that the same scaling could be used in the calculation of all roots. The possibility of using the same scaling had the effect of raising the accuracy of the results, and this is very important in practice.

The calculation of the complex roots by the proposed method was rather rapid; in the example given, about 20 min were needed for the calculation.

The time used in the preliminary calculations for the proposed method was completely compensated for by the time needed for adjusting the apparatus and changing the scaling in the other methods. Moreover, the possibility of finding all the complex roots at once, without any readjustments of the apparatus, justifies this preliminary calculation and testifies to the advantage of using this method. As we pointed out, the amount of equipment needed for the method described is considerably less than in the other methods.

Mechanical Synthesis of Compensating Devices by Means of Adaptive Systems, O. A. Charkviani and V. K. Chichinadze, pp. 160–168.

The results of mechanizing the synthesis of certain automatic control devices by means of adaptive systems are stated. An adaptive system searches for the structure and for the parameters of the synthesized device and stops the search after they are determined.

Volume 23, Number 3, March 1962

Automatic Optimizer for the Search for the Smallest of Several Minima (Global Optimizer), I. N. Bocharov and A. A. Fel'dbaum, pp. 260–270.

Possible principles for constructing a scheme for global optimization are considered, and the development of such a scheme is described. Results are given of tests of an experimental model.

Self-Oscillations in Sampled-Data Extremum Systems, A. V. Netushil, pp. 271–279.

The dependence of the mode of extremum scanning in a sampled-data automatic system on the static characteristic of the system to be controlled is investigated graphically. The conditions under which self-oscillations arise are derived. Examples of self-oscillations which arise in single-channel and two-channel optimizers for different scanning modes are considered.

Extremum automatic systems recently gained widespread use in the most diverse technical devices where it is necessary to maintain the extremal value of a certain quantity to be controlled or computed in the presence of various uncontrollable external disturbances which act on the system.

The maintenance of the extremum (minimum or maximum) value of the controlled variable y can be secured by acting on a series of controllable quantities $x_1, x_2, \ldots, x_n$:

$$y = f(x_1, x_2, \ldots, x_n) \tag{1}$$

In dependence on the uncontrollable external disturbances, this function can assume different forms; however, there is always an explicitly defined extremum, which is determined by the condition

$$\frac{\partial y}{\partial x_i} = \frac{\partial y}{\partial x_2} = \ldots = \frac{\partial y}{\partial x_n} = 0 \tag{2}$$

In investigating various extremum systems, the analysis is most often limited to parabolic characteristics, which involve the simplest calculations, since, in this case, all the partial derivatives are described by the linear dependence

$$\frac{\partial y}{\partial x_i} = \sum_{k=1}^{n} a_{ik} x_k \tag{3}$$

The present article is concerned with the peculiarities observed in deviations of the characteristic of the system to be controlled from a parabolic characteristic in the case where a sampled-data system is used for maintaining the extremum value of the controlled variable. In the analysis, it is considered that the pulse repetition period is sufficiently large for the transient processes in the controlled system to be completed before the beginning of the next cycle.

Optimal Transfer Number for a Reducer in High-Speed Servosystems, G. A. Nadzhafova, pp. 309–314.

Maximum speed of response in automatic control systems, including servosystems, is achieved when the systems are designed on the basis of methods derived from the theory of optimal control. The synthesis of automatic systems with an optimal speed of response reduces to choosing the corresponding actuating power mechanism and the optimal control section. Under these conditions we often neglect the choice of the reducer transfer number, because we proceed from the condition requiring that the maximum speed of response criterion must be satisfied.

A known device for choosing the so-called optimal transfer number of a reducer (this device is widely used in the theory and practice of designing electric drives) is based solely on the condition that optimal acceleration be assured. In another paper an expression is given for choosing the reducer transfer number which is optimal with respect to speed of response for a reducer which does not have bounded coordinates. The problem of using the motor to the limit is not posed there. These devices do not take into account the specific features of servosystems which are designed to reproduce an arbitrary law governing the variation of a specified input. As a result, the maximum possibilities of the actuating mechanism are not fully used. Servosystems thus retain a certain reserve with respect to speed of response, and this reserve can be realized by choosing the optimal value of the reducer transfer number.

In this paper we study problems involved in the choice of an optimal value for the reducer transfer number and the excitation flux for the motor in an electrical servosystem. In contrast to another study, this paper investigates a servosystem that does not operate in a short-duration repetitive mode. Therefore, the choice of the optimal transfer number for the reducer is not associated with a consideration of the limitations imposed on the actuating mechanism by the conditions which govern the heating of the electric motor.

The special operating features and the characteristic for the limitations imposed on the actuating mechanism of the investigated servosystem are cited in other papers. In order to choose the optimal value of the transfer number for the reducer and the excitation flux of the motor, we make use of the method of isochrones.

Conclusions:

1) The methods and formulas cited in the literature for choosing the optimal reducer transfer number do not satisfy the condition requiring maximum speed of response in response of automatic control systems and servosystems.

2) The formulas derived in this paper permit us to determine an optimal value of the reducer transfer number which assures a substantial increase in the speed of response of automatic control system and servosystems.

3) The proposed method for selecting the reducer transfer number is universal, results in an extremal value of the transfer number, and covers methods previously treated in the literature as particular cases.

Experiments on Machine Learning to Recognize Visual Patterns, É. M. Braverman, pp. 315–327.

An algorithm for machine learning to distinguish visual patterns, without using any specific features of those patterns being learned by the machine at a given moment, is described. The algorithm is based solely on the hypothesis of "compactness" of the patterns which, as assumed by the author, is valid for a large class of patterns. The results of an experimental study of the proposed algorithm are presented.

The patterns learned by the machine during the experiment were taken to be the patterns of the numerals 0, 1, 2, 3, and 5. The numerals were placed on a 6 × 10 field; altogether 796 numerals were prepared. Each numeral had a standard vertical dimension (10 cells) and was placed approximately in the center of the field. (We note that if we remain within the framework of the experiment described, in which at each exposure the machine "sees" only a single object, the requirements of centering and standardizing the object with regard to the dimensions is not in principle a limitation, since we can easily conceive of an automatic device which would transform the object to a centered and standarized one.) In learning, about a quarter of the numerals were used (202); the rest were used to test the results of learning.

The basic experimental results are given in Tables 4, 5, and 6. The columns of the tables give the number of the variants, the range of variation of any particular index of efficiency of the algorithm used, and the mean values of these indices, whereas the rows of each table correspond to differing indices of efficiency. Table 4 relates to the initial algorithm, Table 5 to the improved algorithm with $K = 2$, Table 6 to the improved algorithm with $K = 5$.

Using the initial algorithm, we succeeded in obtaining a mean reliability of recognition of 76% with a relatively small memory capacity in the machine, requiring, after learning, less than 3000 bits (to carry out the fourth part of the algorithm it is necessary only to retain the old and new tables and the coefficients of the separating planes). With paralleling of the five variants (1–5) the actual reliability of recognition obtained was 88.4% (calculated 90.4%). Thus without increasing the quantity of objects during learning, it was possible to increase sharply the reliability of recognition, at the expense, it is true, of increasing the required memory capacity of the machine.

The improved algorithm, through its more organized construction of the planes, sharply increases the efficiency. With $K = 5$, already one variant gave up to 90% reliability of recognition, whereas the required memory capacity was decreased to 1500–2000 bits. In this case also practically full independence of the variants was obtained. Thus with the seven variants (11–17) paralleled the actual reliability of recognition—98.5%, whereas calculation gives 98.3%.

Thus the machine effectively learned to recognize the patterns used in the experiment—the numerals 0, 1, 2, 3, and 5. As already mentioned, no information on the properties of these patterns was contained in the program, and, therefore, this machine is in principle capable of learning to distinguish a broad class of other fairly simple patterns.

Analysis of Reliability of Systems with Fault Signaling, V. A. Zhozhikashvili and A. L. Raikin, pp. 352–357.

Indices of reliability of systems are given when the probabilities of the system in use at any arbitrary time instant have been taken into account; the fact that the occurrence of some faults is signaled is also taken into account. Various kinds of operational servicing of the system are examimed. An example is given to illustrate the proposed procedure.

Conclusions:

1) The reliability index of a system when checking and signaling of faults takes place is obtained by extending the usual meaning of reliability of a continually operating system. The probability of failure is now also related to its probability of being in use.

2) Similar indices may prove useful when determining or selecting methods of increasing the reliability of a system or its operation policy.

3) From the examination of the expression given, one observes that in an actual example the average value between successive failures of the system also depends on the transition intensity of the system to the used state, and also on the replacement intensity of the regularly checked faults. In the limit, by making the average repair time approach zero, the maximum average time between failures can be obtained, the latter depending only on the system's transition intensity to its state of use and the intensity of occurrence, in the system, of faults which are not being regularly checked.

Volume 23, Number 4, April 1962

Relations between Adjoints Corresponding to Elements of a Determinant and Their Application to Invariance Theory, V. D. Vershinin, pp. 401–405.

The existence of a relationship between the adjoints of a determinant is shown. Thence a formula can be deduced enabling one to evaluate the determinant from its $n - 1$ adjoints. The mutual dependence of variations of elements is investigated when the value of the determinant remains constant.

Mapping the Movement of a Digital Servosystem on a Multiplane Phase Surface, V. P. Strakhov, pp. 424–435.

This paper demonstrates the possibility of analyzing digital servosystems by mapping their dynamics on a mulitiplane phase surface. A study is made of the behavior of such systems when various typical nonlinearities are present; certain parameter relationships are cited which determine the quality of the transient response.

Summary:

1) The movement of a digital servosystem can be mapped on a multiplane phase surface. Such a representation makes it possible to analyze the transient response and to perform the computation of the system parameters.

2) The behavior of a digital servosystem near the equilibrium position is analogous to that of the conventional relay systems.

3) The relationships between the system parameters which are cited in this paper make it possible to relate the transient response quality to the values of the parameter for the analog section of the system and to the characteristic of the feedback pickup unit which performs the quantization of the angular or linear displacement.

4) An analogous method of investigation can be used when the digital servosystem contains nonlinear elements with more complex characteristics that cause undamped oscillations. Under these conditions, it is possible to derive expressions that determine the selection of system parameters on the basis of the conditions governing the absence of periodic movement.

Canonical Method of Contact Network Synthesis, A. Sh. Blokh, pp. 455–459.

The conductance function of a contact network consisting of contacts $x_1, x_2, \ldots, x_n$ may be given as a function of the integral intermediate parameter s, depending in turn on the $x_1, x_2, \ldots, x_n$. This prescription of the conductance function is, in many cases, more convenient and natural than the traditional form using Boolean functions. Usually the parameter s is found directly from the verbal description of the operating conditions of the contact network and characterizes the invariant properties of the network. Frequently it is more convenient to introduce several intermediate parameters $s_1, s_2, \ldots, s_m$.

In the present article, the canonical method of contact network synthesis is studied for the case of intermediate parameters. Intermediate parameters were first considered in another paper, in connection with a matricial synthesis method. The introduction of intermediate parameters reduces the volume of the numerical tree, which leads to a reduction in machine time and permits functions with a larger number of variables to be realized in EDC.

We first note that the conductance of a $(1,k)$ contact network $x_1, x_2, \ldots, x_n$ is defined by the integral function $N = f(x_1, x_2, \ldots, x_n)$ considering that if $f(\alpha_1, \alpha_2, \ldots, \alpha_n) = \nu$, the conductance between the input and the νth output with $x_1 = \alpha_1$ is equal to unity, and if $f(\alpha_{,1}\ \alpha_2, \ldots, \alpha_n) \neq \nu$, the conductance is equal to zero. If N is a single-valued function, we term the outputs separated. The conductance of a (p, q) network is also defined as a function of $N = f(r, x_1, x_2, \ldots, x_n)$, considering here that if for $r = i$, $x_j = \alpha_j$, we have $f(i, \alpha_1, \alpha_2, \ldots, \alpha_n) = \nu$, the conductance between the ith input and the νth output is equal to unity; if $f(i, \alpha_1, \alpha_2, \ldots, \alpha_n) \neq \nu$, this conductance is equal to zero.

Harmonic Linearization of Nonlinear Inertial Components of Automatic Systems, E. D. Garber, pp. 484–487.

The harmonic linearization of certain nonlinear inertial components of automatic control systems is carried out.

Sensitivity of Hydraulic Nozzle-Flapper Amplifiers, I. M. Krassov, L. I. Radovskii, and B. G. Turbin, p. 491–493.

The sensitivity of hydraulic nozzle-flapper amplifiers under different operating conditions is analyzed. The basic factors to be taken into account in designing the amplifier and determining the parameters which would make it possible to secure the highest sensitivity under the assigned conditions are presented.

BULLETIN OF THE ACADEMY OF SCIENCES USSR, GEOPHYSICS SERIES (*Izvestiia Akademii Nauk SSSR, Seriia Geofizicheskaia*). Published by American Geophysical Union, Washington, D. C.

Number 3, March 1962

Determination of Some of the Spectral Features of Layered Media, L. L. Khudzinsky, pp. 195–203.

Consideration is given to an approximate method for determining the frequency characteristics of reflection for heterogeneous layers at near normal incidence and the spectra of the reflected waves corresponding to these layers.

Conclusions:

1) A method is suggested for the determination of the frequency characteristic of reflection from a heterogeneous layer with arbitrary distribution of impedance and the spectrum of the reflected wave recorded at the surface.

2) The frequency characteristic of reflection from a heterogeneous layer coincides approximately with the spectrum of the density $\kappa(t_0)$ of the reflection factor or with the spectrum of the previously differentiated logarithm of the impedance.

3) Two methods of generating pulses of special shape used for approximate determination of the frequency characteristics of reflection from a heterogeneous layer are considered.

4) The multiple pulse generator enables one to obtain periodically repeated trains of narrow square pulses. The generator can be used to determine the frequency characteristic of reflection of a heterogeneous layer consisting of several contacting homogeneous layers. It permits variation of the thickness of one or more of the layers in the heterogeneous layer and of the values of the reflection factors from the sharp interfaces corresponding to this layer.

5) Graphic depiction of the logarithm of impedance is essential for determination of the frequency characteristic of reflection of a heterogeneous layer in which impedance is arbitrarily dependent on depth. The trace of $\ln \gamma(t_0)$ produced by